1st Week

22 Şubat 2023 Çarşamba 11:26

Office hour : Tue : 15:30 – 16:30
→ Wed : 13:30 – 14:30

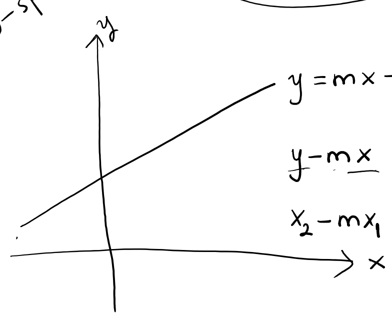
Registration Key : mat104esp23

System of Linear Eqns ⇄ Matrices

Systems of Linear Equations

line

2D-space



$y = mx + n$

$y - mx = n$

$x_2 - mx_1 = n$

x, y

x, y, z

linear equations

unknowns / variables

$x_1, x_2, \dots, x_n$

A single linear eqn → $\boxed{a_1 x_1 + a_2 x_2 + \dots + a_n x_n = b}$

coefficients → $a_1, a_2, \dots, a_n \in \mathbb{R}$

the result / the constant term $b \in \mathbb{R}$

Ex $3x_1 - 2x_2 + 5x_3 = 21$

Ex $3x_1^2 - 5x_2 + 3x_3 = 20$

not linear !

- In a system; eqns use the same variables.
- the solution of the system should satisfy all the eqns in the system. → (intersection point)

the coefficient of the j^{th} variable in the i^{th} eqn $= \boxed{a_{ij}}$

eqn, # variable

$$\left[\begin{array}{l} a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1 \\ a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2 \\ \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m \end{array}\right]$$

→ 1st eqn → represents a line n-D

→ 2^{nd} eqn → 〃

→ m^{th} eqn →

→ $m \times n$ system of linear eqns.

\# eqn → # variable

→ $m \times n$ system of linear eqns.

m equations n unknowns

Solution $(x_1, x_2, \dots, x_n)$ → an ordered n-tuple.
which satisfies all the eqns in the system.

Ex

$$\begin{cases} x_1 + 3x_2 - x_5 = 3 \\ x_1 - 4x_4 + x_6 = 5 \\ 2x_2 + 3x_3 - x_5 = 1 \end{cases}$$

3 eqns, 6 unknowns

find the intersection pt of 3 lines on 6-D space

$x_1 + 3x_2 + 0x_3 + 0x_4 - x_5 + 0x_6 = 3$

3×6

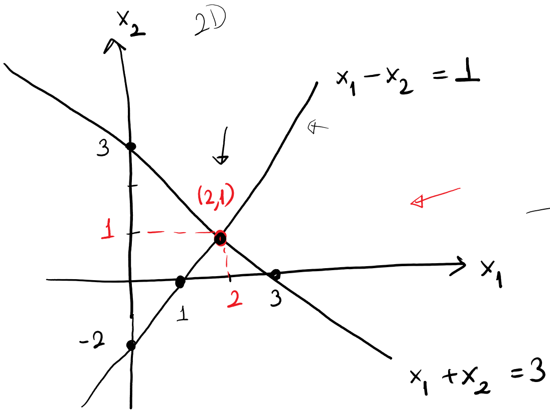
Find $(x_1, x_2, x_3, x_4, x_5, x_6)$ → a single point

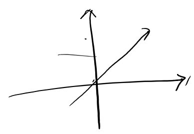
6-D

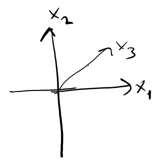
2D-space 2x2 systems

Ex

$$\begin{cases} x_1 + x_2 = 3 \\ x_1 - x_2 = 1 \end{cases}$$







① (x_1, x_2)

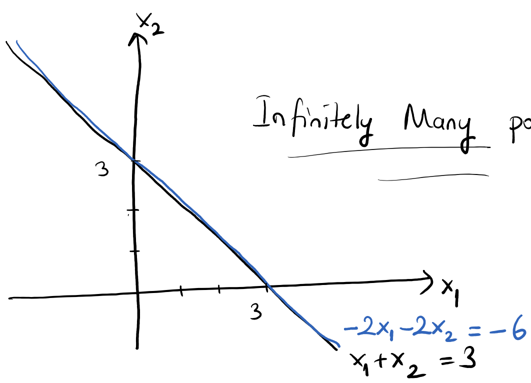
$2x_1 = 4 \Rightarrow x_1 = 2$
$2x_2 = 2 \Rightarrow x_2 = 1$ } $(2,1)$

Solution Set = $\{(2,1)\}$ → unique solution.

Ex

$x_1 = 1 \quad x_2 = 4$

$$\begin{bmatrix} x_1 + x_2 = 3 \\ -2x_1 - 2x_2 = -6 \end{bmatrix}$$



Infinitely Many points shared.

② $x_1 + x_2 = 3$

$x_2 = r \in \mathbb{R}$ → free variable (independent)

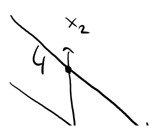
$x_1 = 3 - r$ → dependent variable (it depends on the value r)

Solution Set = $\{(3-r, r) : r \in \mathbb{R}\}$

x_2 $r = 2$ $x_1 = 3 - 2 = 1$ $(1,2)$ ✓
$r = 3$ $x_2 = 3$ $x_1 = 0$ $(0,3)$ ✓
...

infinitely many solutions.

③ $x_1 + x_2 = 3$ }



they don't share any points!

3

$x_1 + x_2 = 3$

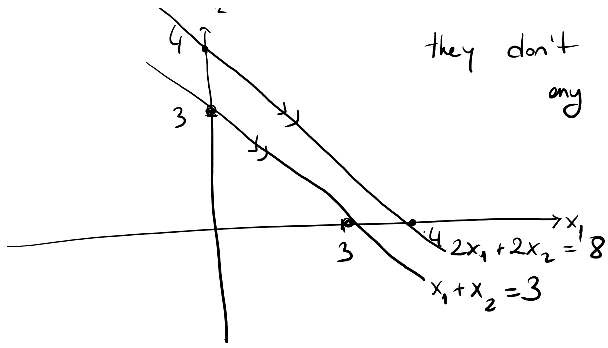
$2x_1 + 2x_2 = 8$

$2x_1 + 2x_2 = 6$

$- \; 2x_1 + 2x_2 = 8$

$0x_1 + 0x_2 = -2$

$\boxed{0 = -2}$ ‼ impossible!



they don't share any points!

⇒ This system has NO solution!

For any n-dim space, for any <u>m x n system</u>;

the only 3 possibilities for the solution set

1) unique solution $\{(x_1, x_2, \ldots, x_n)\}$ real values

2) infinitely many solutions $\{$ at least one of x_i's will be a free variable $\}$

3) no solution

while eliminating your system

→ $0x_1 + \ldots + 0x_n =$ nonzero number!

impossible!